CHARLES DARWIN UNIVERSITY

Family Name	
Given Names	
Student Number	
Teaching Period	Semester 1, 2016

FINAL EXAMINATION	**DURATION**
ENG421 – Digital Signal Processing	Reading Time: **10** minutes Writing Time: **120** minutes

INSTRUCTIONS TO CANDIDATES

1 Answer all questions

2 This exam constitutes 50% of the total marks for this unit

3 Total number of marks of this exam: 16

Question 1 is worth 3 marks

Question 2 is worth 3 marks

Question 3 is worth 2 marks

Question 4 is worth 2 marks

Question 5 is worth 3 marks

Question 6 is worth 3 marks

EXAM CONDITIONS

You may begin writing from the commencement of the examination session. The reading time indicated above is provided as a guide only.

This is a RESTRICTED OPEN BOOK examination

Any non-programmable calculator is permitted

No handwritten notes are permitted

Any hard copy, unannotated English dictionary is permitted

ADDITIONAL AUTHORISED MATERIALS	**EXAMINATION MATERIALS TO BE SUPPLIED**
No additional printed material is permitted	1 x 20 Page Book

THIS EXAMINATION IS PRINTED DOUBLE-SIDED.

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Question 1 (3 marks)

The following difference equation describes a linear time-invariant filter:

$$y[n] = 1.25 \cdot y[n+1] - 1.25 \cdot x[n-1] + x[n]$$

Question 1.1 (1 mark)

Determine the system transfer function in the z-domain for this filter. Determine all poles and zeros of this filter.

Question 1.2 (1 mark)

Determine $\left| H(e^{j\hat{\omega}}) \right|^2$ for all $\hat{\omega}$.

Question 1.3 (1 mark)

Determine the output of the filter, if the input to the filter is

$$x[n] = 5 - 3\sin(\frac{\pi}{3} \cdot n) - 2\cos(\frac{\pi}{2} \cdot n)$$

Question 2 (3 marks)

A system S is linear and time invariant. The following inputs x[n] result in the following outputs y[n]:

$$x[n] = \delta[n] - \delta[n-1] \quad \rightarrow \quad y[n] = 2\delta[n] - 2\delta[n-4]$$

$$x[n] = \cos\left(\frac{\pi \cdot n}{2}\right) \quad \rightarrow \quad y[n] = 0$$

$$x[n] = \cos\left(\frac{\pi \cdot n}{3}\right) \quad \rightarrow \quad y[n] = 6 \cdot \cos\left(\frac{\pi \cdot n}{3} - \frac{\pi}{2}\right)$$

Question 2.1 (1 mark)

Determine the output of the system if the input is:

$$x[n] = 3\delta[n] - 3\delta[n-3]$$

Question 2.2 (1 mark)

Determine the output of the system if the input is:

$$x[n] = 5\sin\left(\frac{\pi \cdot (n-2)}{3}\right)$$

Question 2.3 (1 mark)

Determine the output of the system if the input is:

$$x[n] = 9\sin\left(\frac{\pi n}{2} + \frac{5\pi}{6}\right)$$

Question 3 (2 mark)

Let x[n] be the complex exponential:

$$x[n] = 7e^{j(0.22\pi n - \pi)}$$

y[n] is the output of the system which is described by the difference equation:

$$y[n] = x[n] - 2x[n-1] + x[n-2]$$

It is now possible to describe the output y[n] of the system in the form of:

$$y[n] = Ae^{j(\hat{\omega}_0 \cdot n + \phi)}$$

Question 3.1 (1 mark)

Determine the numerical value of A

Question 3.2 (1 mark)

Determine the numerical value of ϕ.

Question 4 (2 marks)

A linear time invariant system has the following frequency response:

$$\hat{H}(\omega) = (e^{-\pi} - e^{-j3\hat{\omega}}) \cdot (1 - e^{-j\frac{\pi}{2}} \cdot e^{-j3\hat{\omega}}) \cdot (1 + e^{-j\frac{\pi}{2}} \cdot e^{-j3\hat{\omega}}) \cdot$$

Question 4.1 (1 mark)

Determine the impulse response h[n] of this system.

Question 4.2 (1 mark)

The input to the system is:

$$x[n] = 4 \cdot \sin(\frac{\pi}{6} n + \frac{\pi}{3}) - 2 \cdot \delta[n-1] + 1$$

Determine the output y[n] for $-\infty < n < \infty$.

Question 5 (3 marks)

A system has the following transfer function in the z-domain:

$$H(z) = \frac{(1 - z^{-1}) \cdot (e^{-j\frac{\pi}{2}} - z^{-1}) \cdot (e^{j\frac{\pi}{2}} - z^{-1})}{(2 - \cdot e^{j\frac{\pi}{3}} z^{-1}) \cdot (2 - e^{-j\frac{\pi}{3}} z^{-1})}$$

Question 5.1 (1 mark)

Derive the difference equation describing this system, with x[n] as input and y[n] as output.

Question 5.2 (1 mark)

Determine all poles and zeros of this system. Make a sketch of the pole-zero plot.

Question 5.3 (1 mark)

If the input of the system is of the form

$$x[n] = A \cdot e^{j\phi} \cdot e^{j\hat{\omega}},$$

for what values of $-\pi \le \hat{\omega} \le \pi$ will y[n] = 0?

Question 6 (3 marks)

A system is defined by the following z-domain transfer function:

$$H(z) = (1 + e^{j\frac{\pi}{2}} z^{-1}) \cdot (1 + e^{-j\frac{\pi}{2}} z^{-1}) \cdot (1 + e^{-j\pi} z^{-1}) \cdot (1 + z^{-1}) \cdot$$

Question 6.1 (1 mark)

Determine the difference equation describing this system, with x[n] as input and y[n] as output.

Question 6.2 (1 mark)

Determine the frequency domain response of this system and derive two simple formulas (without complex terms and without square roots) for the magnitude versus $\hat{\omega}$ and the phase versus $\hat{\omega}$. Sketch the magnitude versus $\hat{\omega}$ and the phase versus $\hat{\omega}$ for $-\frac{\pi}{2} \le \hat{\omega} \le \frac{\pi}{2}$.

Question 6.3 (1 mark)

Determine the output of the system y[n] if the input to the system is
$x[n] = \cos(\pi \cdot \frac{n}{4} + \frac{\pi}{3})$.

Attached Materials

SHORT TABLE OF z-TRANSFORMS

	$x[n]$		$X(z)$
1.	$ax_1[n] + bx_2[n]$	$\Longleftrightarrow$	$aX_1(z) + bX_2(z)$
2.	$x[n - n_0]$	$\Longleftrightarrow$	$z^{-n_0}X(z)$
3.	$y[n] = x[n] * h[n]$	$\Longleftrightarrow$	$Y(z) = H(z)X(z)$
4.	$\delta[n]$	$\Longleftrightarrow$	1
5.	$\delta[n - n_0]$	$\Longleftrightarrow$	z^{-n_0}
6.	$a^n u[n]$	$\Longleftrightarrow$	$\dfrac{1}{1 - az^{-1}}$

PROCEDURE FOR INVERSE z-TRANSFORMATION $(M < N)$

1. Factor the denominator polynomial of $H(z)$ and express the pole factors in the form $(1 - p_k z^{-1})$ for $k = 1, 2, \ldots, N$.
2. Make a partial fraction expansion of $H(z)$ into a sum of terms of the form

$$H(z) = \sum_{k=1}^{N} \frac{A_k}{1 - p_k z^{-1}} \quad \text{where} \quad A_k = H(z)(1 - p_k z^{-1})\Big|_{z=p_k}$$

3. Write down the answer as

$$h[n] = \sum_{k=1}^{N} A_k (p_k)^n u[n]$$